# Ppts via categorical decompositions + wigglyhedra — 18/11/25

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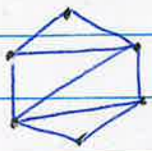
**Combinatorics** ↓ **Geometry** ↑ **Categories**

**Triangulation:**



Classical

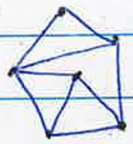
+ associahedron

**Convex:**

Arcs ⟷ Objects
+ HN filtration

BDL

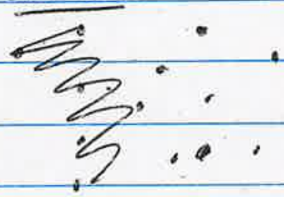
**Ppt:**

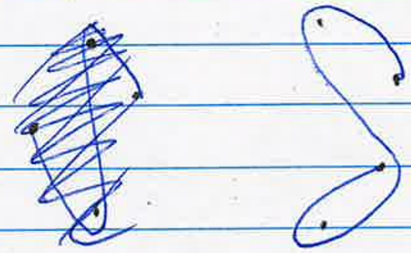


RSS

+ ppt polytope

**Generic:**

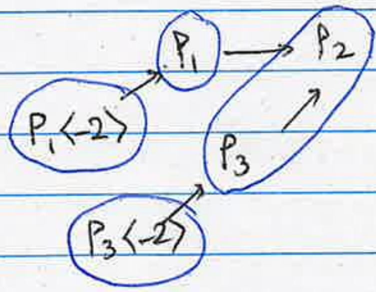




**(wiggly) ppt**

+ ?

**Intermediate:**




**(wiggly) ppt**

BP

+ wigglyhedron

**Aligned:**

Arcs ⟷ objects

⟷ $P_1\langle -2\rangle \to P_1 \to P_2$, $P_3\langle -2\rangle \to P_3 \to P_2$

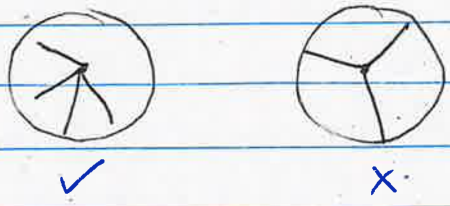
$$\begin{array}{ccc} & & P_1 \to P_2 \\ & \nearrow & \\ P_1\langle -2\rangle & & P_3 \nearrow \\ & & \nearrow \\ & P_3\langle -2\rangle & \end{array}$$

KS

**Def** (ppt). A pointed pseudo-triangulation (ppt) on a point set is a collection of wiggly segments that is:

1. pairwise non-crossing

2. pointed @ each vertex:



**Thm** [Rote-Santos-Streinu (2003)] For a generic configuration/point set, there is a polytope whose 1-skeleton is the flip graph of ppts on that point set
(In convex position, recover an associahedron.)

**Rmk**: Construction uses rigidity theory.

$\boxed{\text{"Wigglyhedra" - Math. Z.}}$

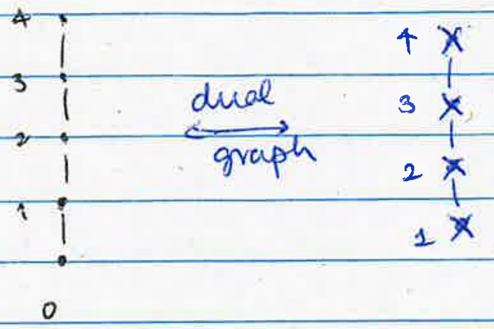
**Thm** [B.-Piland, 2025] For an aligned config, there is a polytope whose 1-skeleton is the flip graph of ppts on that set — the wigglyhedron.

**Rmk**:
- Construction only combinatorial
- This is, in some sense, the easiest + hardest case as opposed to convex position: easiest + easiest.

**Def** (wiggly arc): A curve / isotopy that starts and goes monotonically in one direction w/o self intersection + going through points to reach another, + arbitrary close to straight line segment.

## Categorical background

Consider a set of $(n+1)$ distinct points (aligned) + segments




type $A_n$ graph.

$\downarrow$ gives rise to

2CY Khovanov-Seidel category $\mathcal{C}_n$.
and a "heart" subcategory
$\heartsuit_n \subset \mathcal{C}_n$.

Features

- $\mathcal{C}_n$ is triangulated (objects are "complexes")
- $\heartsuit_n$ has $n$ simple objects $P_1, P_2 \dots P_n \longleftrightarrow$ simple +ve roots $\alpha_i$
- $\heartsuit_n = \langle P_1, \dots, P_n \rangle$, and Grothendieck gp $\simeq$ root lattice.

i.e if $X \in \heartsuit_n$ then $[X] = \sum n_i \alpha_i$ counts # $P_i$

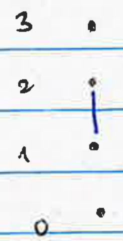
Recall: $S_n \circlearrowright$ root lattice. This action "lifts" or categorifies.

$$\begin{array}{ccc} S_n & \circlearrowright & \text{root lattice} \\ \uparrow & & \uparrow \\ B_n & \circlearrowright & \mathcal{C}_n \end{array}$$

(*) Does not preserve $\heartsuit_n$!

[Khovanov-Seidel + Seidel-Thomas etc early 2000-ish]

## Khovanov-Seidel correspondence

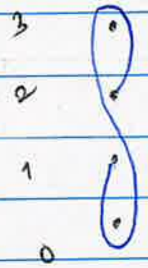


$= P_2$

Given arc $\alpha$, write it as

~~$b(\alpha)$ for some~~ $\alpha = b(\alpha_i)$ for some $b \in B_n$.

Then the corresponding object is $b(P_i)$.



$$= \sigma_3^2 \sigma_1^2 (P_2) = \boxed{\begin{matrix} P_1\langle -2\rangle \\ P_3\langle -2\rangle \end{matrix}} \longrightarrow \boxed{\begin{matrix} P_1 \\ \oplus \\ P_3 \end{matrix} \longrightarrow P_2}$$

~~We have~~ Objects have "Jordan-Hölder" filtration
$\rightarrow$ break into $\{P_i\}$.

$\exists$ another filtration : cohomology filtration!
objects break up into elements of $\heartsuit_n$.

Fact : Wiggly arcs $\longleftrightarrow$ $\heartsuit_n$

Thm : $\alpha \rightsquigarrow$ cut along U-turns $\equiv$ cohomology filtration.

$$X \mapsto \sum_{\alpha \in \heartsuit_n} n_\alpha \alpha$$

<u>Note</u>: Compatible arcs satisfy the (wiggly) ppt condition
+ extra condition.

So, consider simplicial complex of (wiggly) ppts $K(P)$

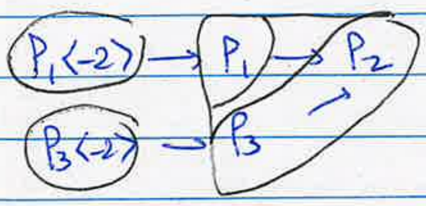
$X \in \mathcal{C} \rightsquigarrow$ pt on bdry of $K(P)$

$\boxed{\text{arXiv:2509.13912}}$

<u>Deform</u>!

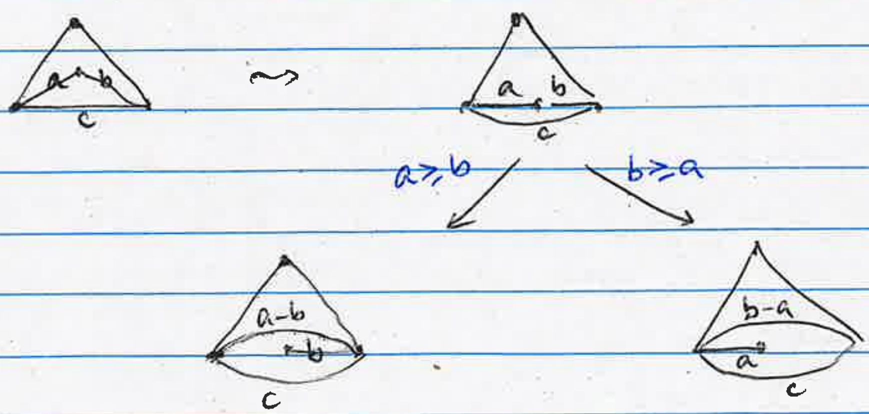
<u>Thm</u> [BDL]: Any point set gives a <u>stability function</u> on $\heartsuit_n$
Objects then have a HN filtration whose pieces are semistable objects in $\heartsuit_n$.

<u>Thm</u> [BDL]: $\alpha \mapsto$ cut along bends
$\equiv$ HN filtration




<u>Thm</u> [BDL]: Simplicial complex of wiggly ppts is pure + spherical.

<u>Picture</u>:




$\boxed{\text{Questions}}$